

# **Predicting the maximum of 363-day smoothed highest 3-hourly *aa* index in 3-day-interval with the preceding minimum of 11-year cycle**

**Zhanle Du**

Key Laboratory of Solar Activity, National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China.

Correspondence to: Z. L. Du  
(zldu@nao.cas.cn)

## Abstract

Predicting the strength and peak time of geomagnetic activity for the ensuing cycle 25 is important in space weather service for planning future space missions. This study analyzed the **highest ( $aa_H$ ) and lowest  $aa$  index ( $aa_L$ ) from 24 three-hourly values in 3-day-interval, smoothed by 363 days (121 points) to mimic the 13-month smoothing. It is found that the maximum ( $aa_{Hmax}$ ) of  $aa_H$  is well correlated to both the preceding minimum ( $aa_{Hmin}, r = 0.85$ ) of  $aa_H$  and the preceding minimum ( $aa_{Lmin}, r = 0.89$ ) of  $aa_L$  for the 11-year solar cycle. Based on these relationships the strength of geomagnetic activity for the ensuing cycle is predicted to be  $aa_{Hmax}(25) = 85.5 \pm 6.9$  (nT), about 32% stronger than that of cycle 24. This value is equivalent to the Ap index of  $Ap_{max}(25) = 56.0 \pm 4.8 \pm 1.2$  (nT). The maximum ( $aa_{Lmax}$ ) of  $aa_L$  is also found to be well correlated to the preceding  $aa_{Hmin}$  ( $r = 0.80$ ). The solar activity is much better correlated to the strong geomagnetic activity ( $aa_{Hmax}, r = 0.79$ ) than to the weak one ( $aa_{Lmax}, r = 0.37$ ). The rise time of  $aa_{Hmax}$  ( $T_{Hr}$ ) is found to be weakly anti-correlated to the following maximum ( $aa_{Hmax}$ ),  $r = -0.42$  at the 84% confidence level. Using this correlation and the predicted  $aa_{Hmax}(25)$ , one can roughly estimate the rise time,  $T_{Hr}(25) = 5.2 \pm 2.0$  (years), implying that the geomagnetic activity for the ensuing cycle 25 would peak around August 2025 $\pm$ 2.0 (years).**

## 1 Introduction

Studying and predicting geomagnetic activities are important in both geophysics and space weather. Severe geomagnetic activities may cause intense geomagnetic storms (Gonzalez et al., 1989, 1994; Chen et al., 2019), leading to disruptions in communication and deviations of spacecrafts. With the current solar cycle 24 approaching its end, satellite and spacecraft-related departments want to know the strengths of both solar and geomagnetic activities in the ensuing cycle 25 for planning future space missions.

Among various indices to quantitatively describe the geomagnetic activity, the  $aa$  index (Mayaud, 1972), derived from the 3-hourly K indices at two near-antipodal mid-

latitude stations in England and Australia, is the longest time series (since 1868) and has been widely used for analyzing long-term trends in the global geomagnetic activity (Russell and Mulligan, 1995; Marat et al., 2017; Du, 2011a; El-Borie et al., 2019) and for analyzing its correlation with both climate change (Cliver et al., 1998; Dobrica et al., 2009; Gavriljeva et al., 2017) and solar activity (Echer et al., 2004; Prestes et al., 2006; Lukianova et al., 2009; Du, 2011b,c; Du and Wang, 2012; Singh and et al., 2019). The minimum  $aa$  index ( $aa_{\min}$ ), at or near the minimum of the solar cycle, has been widely used in predicting the maximum amplitude of the sunspot cycle ( $R_m$ ), the so-called Ohl's precursor method (Ohl, 1979; Brown and Williams, 1969; Du et al., 2009). But it is seldom used to directly predict the maximum  $aa$  index ( $aa_{\max}$ ) of an ensuing cycle.

The planetary geomagnetic index  $Ap$  (Bartels, 1963) available since 1932, derived from the average of the measurements at 13 observatories around the globe, is a daily measure of the response of geomagnetic field to variations in the interplanetary magnetic field (IMF) and the solar wind (Li, 1997; McPherron, 1999; Tsurutani et al., 2006). It is the main global magnetic index forecasted by government agencies (McPherron, 1999). Most works on forecasting geomagnetic activity have been over short intervals, on the order of hours or days (McPherron, 1999; Abunina et al., 2013). In the earlier years, Kane (1988) pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the daily, monthly and annual values of  $Ap$  and  $aa$  indices. Gordon (2015) demonstrated that long-term geomagnetic activity can only be predicted to within a limited threshold of accuracy due to the irregular trends and cycles in the annual data and nonlinear variability in the monthly series, through analyzing the  $aa$  index.

In this study, we analyze the **highest ( $aa_H$ ) and lowest ( $aa_L$ ) 3-hourly  $aa$  index in each 3 days' interval, smoothed by 363 days (121 points) to mimic the 13-month smoothing. It is found that the maximum ( $aa_{H\max}$ ) of  $aa_H$  is well correlated to both the preceding minimum ( $aa_{H\min}$ ) of  $aa_H$  and the preceding minimum ( $aa_{L\min}$ ) of  $aa_L$  for the 11-year solar cycle, which can be used to predict  $aa_{H\max}$ .**

This study is arranged as follows. The data used in the current work are shown

in Sect. 2. Section 3 is devoted for the results. First, in Sect. 3.1, we analyze the relationships between the maximum of the smoothed highest 3-hourly  $aa$  index ( $aa_{Hmax}$ ) in 3 days and the preceding minima of both the smoothed highest ( $aa_{Hmin}$ ) and lowest ( $aa_{Lmin}$ ) 3-hourly  $aa$  indices in 3 days, followed by a prediction of  $aa_{Hmax}$  for cycle 25. **The relationship between the maximum of  $aa_L$  ( $aa_{Lmax}$ ) and the preceding  $aa_{Hmin}$  is simply analyzed in Sect. 3.2.** In Sect. 3.3, we analyze the relationship between the rise time of  $aa_H$  from  $aa_{Hmin}$  to  $aa_{Hmax}$  and the following maximum for the 11-year cycle, so as to estimate the peak time of geomagnetic activity for the ensuing cycle. Some conclusions are discussed and summarized in Sect. 4.

## 2 Data

In this study, we use the 3-hourly  $aa$  index since 1868 from the International Service of Geomagnetic Indices (ISGI)<sup>1</sup>. For each 3-day-interval, we find the highest  $aa$  index ( $aa_H$ ) and the lowest  $aa$  index ( $aa_L$ ) from 24 values of the 3-hourly  $aa$  indices. Then, both  $aa_H$  and  $aa_L$  are smoothed by 363 days (121 points) to mimic the 13-month smoothing, as shown in Fig. 1 (solid). The (13-month) smoothed monthly mean International sunspot number series ( $R_l$ , Clette et al., 2016) of the second version<sup>2</sup> is used for comparison (dotted).

The parameters used in this study are listed in Table 1, in which  $aa_{Hmin}$  ( $aa_{Hmax}$ ) is the minimum (maximum) of  $aa_H$ ,  $aa_{Lmin}$  ( $aa_{Lmax}$ ) the minimum (maximum) of  $aa_L$ ,  $T_{Hr}$  the rise time of  $aa_H$  from  $aa_{Hmin}$  to  $aa_{Hmax}$ , and  $R_m$  the maximum of  $R_l$  for the 11-year solar cycle. The last row denotes the averages of the parameters.

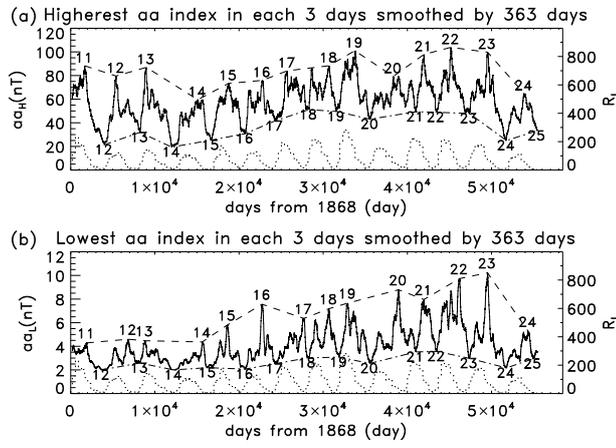
---

<sup>1</sup><http://isgi.unistra.fr/>

<sup>2</sup><http://www.sidc.be/silso/datafiles>

**Table 1.** The minimum ( $aa_{Hmin}$ ) and maximum ( $aa_{Hmax}$ ) of the 363-day smoothed highest 3-hourly  $aa$  index ( $aa_H$ ) in each 3 days, the rise time of  $aa_{Hmax}$  ( $T_{Hr}$ ), the minimum ( $aa_{Lmin}$ ) and maximum ( $aa_{Lmax}$ ) of the 363-day smoothed lowest 3-hourly  $aa$  index ( $aa_L$ ) in each 3 days, and the maximum ( $R_m$ ) of 13-month smoothed sunspot number for solar cycles 11-25.

$n$	$aa_{Hmin}(nT)$	$aa_{Hmax}(nT)$	$T_{Hr}(yr)$	$aa_{Lmin}(nT)$	$aa_{Lmax}(nT)$	$R_m$
11		88.24			4.29	234.0
12	21.41	79.60	3.81	2.00	4.60	124.4
13	31.88	86.92	2.08	2.58	4.47	146.5
14	19.40	59.55	8.47	2.00	4.37	107.1
15	25.31	73.28	5.72	2.19	5.80	175.7
16	30.80	75.90	5.56	2.07	7.56	130.2
17	41.60	84.01	4.01	2.52	6.36	198.6
18	51.13	88.31	6.57	3.02	7.18	218.7
19	49.60	100.99	5.89	3.08	7.66	285.0
20	43.08	79.31	7.54	2.55	8.79	156.6
21	49.35	97.87	2.63	3.52	7.96	232.9
22	48.93	103.89	4.78	3.57	9.73	212.5
23	47.84	99.75	5.92	2.98	10.22	180.3
24	25.02	64.81	5.73	2.17	5.27	116.4
25	33.24			2.94		
Av.	37.04	84.46	5.29	2.66	6.73	179.9



**Fig. 1.** (a) The highest  $aa$  index ( $aa_H$ ) and (b) the lowest  $aa$  index ( $aa_L$ ) in each 3 days, smoothed by 363 days. The numbers in the figure indicate the 11-year solar cycles. The upper dashed and lower dash-dotted lines indicate the maxima and minima, respectively, for the 11-year cycle. The dotted line represents the smoothed monthly mean sunspot number ( $R_i$ ) for comparison.

### 3 Result

The correlation coefficients between the parameters in Table 1 are listed in Table 2 for comparison. It is seen in Table 2 that  $R_m$  is well correlated to  $aa_{Hmin}$  ( $r = 0.84$ ),  $aa_{Hmax}$  ( $r = 0.79$ ),  $aa_{Lmin}$  ( $r = 0.81$ ), and positive correlated to  $aa_{Lmax}$  ( $r = 0.37$ ). It implies that the stronger the solar activity ( $R_i$ ), the higher the geomagnetic activity ( $aa$ ). But the solar activity is much better correlated to the strong geomagnetic activity ( $aa_{Hmax}$ ) than to the weak one ( $aa_{Lmax}$ ), the latter may be due to the high speed solar wind streams which is out of phase with the solar activity cycle (Feynman, 1982; Hathaway and Wilson, 2006). The

**Table 2.** The correlation coefficient ( $r$ ) between parameters  $x$  and  $y$ .

$x/y$	$aa_{Hmin}$	$aa_{Hmax}$	$T_{Hr}$	$aa_{Lmin}$	$aa_{Lmax}$	$R_m$
$aa_{Hmin}$	1.00	0.85	-0.10	0.87	0.80	0.84
$aa_{Hmax}$	0.85	1.00	-0.42	0.89	0.63	0.79
$T_{Hr}$	-0.10	-0.42	1.00	-0.28	0.13	-0.18
$aa_{Lmin}$	0.87	0.89	-0.28	1.00	0.70	0.81
$aa_{Lmax}$	0.80	0.63	0.13	0.70	1.00	0.37

correlation between  $R_m$  and  $aa_{Hmin}$  (or  $aa_{Lmin}$ ) is related to the Ohl's precursor method (Ohl, 1979) for predicting  $R_m$ . Other significant correlations can be used to predict  $aa_{Hmax}$  (Sect. 3.1),  $aa_{Lmax}$  (Sect. 3.2), and  $T_{Hr}$  (Sect. 3.3) in this study.

### 3.1 Relationship between the maximum and preceding minimum

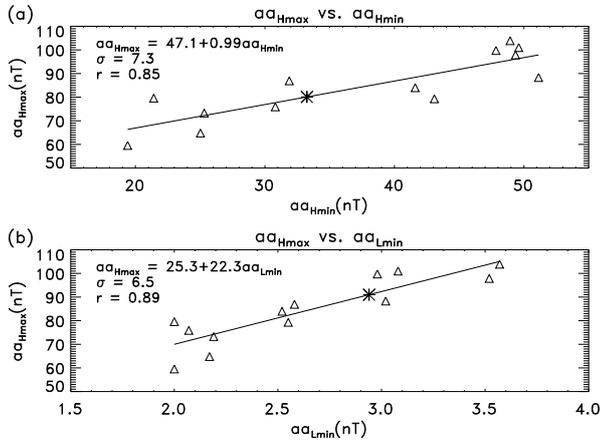
5 It is seen in Table 2 that  $aa_{Hmax}$  is well correlated to the preceding  $aa_{Hmin}$  ( $r = 0.85$ ) and  $aa_{Lmin}$  ( $r = 0.89$ ), as shown in Fig. 2 for the scatter plots of  $aa_{Hmax}$  against  $aa_{Hmin}$  (a) and  $aa_{Lmin}$  (b). The solid line represents a linear fit of  $aa_{Hmax}$  to  $aa_{Hmin}$  ( $aa_{Lmin}$ ) with the least-squares-fit regression equations given by

$$\begin{cases} aa_{Hmax} = 47.1 \pm 7.1 + (0.99 \pm 0.18)aa_{Hmin}, \sigma = 7.3, \\ aa_{Hmax} = 25.3 \pm 9.4 + (22.3 \pm 3.5)aa_{Lmin}, \sigma = 6.5, \end{cases} \quad (1)$$

10 where  $\pm$  represents the  $1\sigma$  deviation and  $\sigma$  the standard deviation of the fitting.

Therefore, the  $aa$  index at the minimum can be used as an indicator to predict the index at the maximum. From the above relationships, one can predict  $aa_{Hmax}$  for cycle  $n = 25$  by substituting the values of  $aa_{Hmin}$  (33.24 nT) and  $aa_{Lmin}$  (2.94 nT) into these equations,

$$\begin{cases} aa_{Hmax1}(25) = 80.1 \pm 7.3(\text{nT}), \text{ from } aa_{Hmin}, \\ aa_{Hmax2}(25) = 91.0 \pm 6.5(\text{nT}), \text{ from } aa_{Lmin}, \end{cases} \quad (2)$$



**Fig. 2.** (a) Scatter plot of  $aa_{Hmax}$  against  $aa_{Hmin}$  (triangles) and the linear fit (solid). (b) Scatter plot of  $aa_{Hmax}$  against  $aa_{Lmin}$  (triangles) and the linear fit (solid).

(labelled by asterisk). As the above relationships have a similar correlation (0.85 and 0.89), we take the prediction of  $aa_{Hmax}(25)$  as the average,

$$\begin{aligned}
 aa_{Hmax}(25) &= \frac{1}{2}[aa_{Hmax1}(25) + aa_{Hmax2}(25)] \\
 &= 85.5 \pm 6.9(\text{nT}).
 \end{aligned}
 \tag{3}$$

It implies that the 363-day smoothed highest 3-hourly  $aa$  index in 3-day-interval during the maximum period of cycle 25 is predicted to be close to the average (84.46 nT) over the past cycles (Table 1), but higher than that (64.81 nT) of cycle 24 by about 32.0%.

It should be pointed out that the above prediction may be an upper estimate for the maximum  $aa$  index as cycle 24 has not completely passed. Although we are not quite sure if the current  $aa_H$  ( $aa_L$ ), 33.94 (3.47) in November 2019, would decrease to a smaller value than that, 33.24 (2.94), used in the current work, there would not be significant variations

in  $aa_{Hmin}$ ,  $aa_{Lmin}$  and the above prediction, as the solar (sunspot) activity ( $R_l$ ) shows a sign to stop decreasing and to oscillate around the minimum in the recent few months.

### 3.2 Relationship between $aa_{Lmax}$ and the preceding $aa_{Hmin}$

It is seen in Table 2 that  $aa_{Lmax}$  is also well correlated to the preceding  $aa_{Hmin}$  ( $r = 0.80$ ), as shown in Fig. 3(a) for the scatter plot of  $aa_{Lmax}$  against  $aa_{Hmin}$ . The linear fitting equation of  $aa_{Lmax}$  to  $aa_{Hmin}$  (solid) is

$$aa_{Lmax} = 2.0 \pm 1.2 + (0.131 \pm 0.030)aa_{Hmin}, \sigma = 1.2. \quad (4)$$

Substituting  $aa_{Hmin}(25) = 33.24$  (nT) into this equation, one can estimate the 363-day smoothed lowest 3-hourly  $aa$  index in 3-day-interval during the maximum period of cycle 25,  $aa_{Lmax} = 6.4 \pm 1.2$  (nT). This value is close to the average (6.73 nT) over the past cycles, but higher than that (5.27 nT) of cycle 24 by 21.1%.

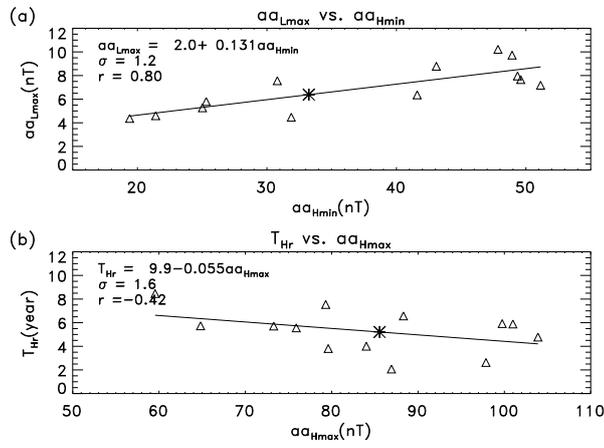
### 3.3 Relationship between the rise time and maximum

At last, in this section, we analyze if the rise time of the geomagnetic index for the 11-year cycle is correlated to the following maximum so that it can be used to estimate the rise time, as the case in sunspot cycle (Waldmeier, 1939).

Figure 3(b) shows the scatter plot of the rise time ( $T_{Hr}$ ) from  $aa_{Hmin}$  to  $aa_{Hmax}$  for the 11-year cycle against its following maximum ( $aa_{Hmax}$ ). It is seen in this figure that  $T_{Hr}$  is weakly anti-correlated to  $aa_{Hmax}$ , with a correlation coefficient of  $r = -0.42$  at a confidence level of about 84%. The linear fitting equation of  $T_{Hr}$  to  $aa_{Hmax}$  is

$$T_{Hr} = 9.9 \pm 3.0 - (0.055 \pm 0.036)aa_{Hmax}, \sigma = 1.6. \quad (5)$$

Using this weak correlation, one can roughly estimate the rise time,  $T_{Hr}(25) = 5.2 \pm 0.4 \pm 1.6$  (years), by substituting the predicted value of  $aa_{Hmax}(25) = 85.5 \pm 6.9$  (nT) in Eq. (3) into this equation, here  $\pm 0.4$  is derived from the uncertainty ( $\pm 6.9$ ) of  $aa_{Hmax}(25)$ . Certainly, this estimate is less reliable than the prediction on the maximum.



**Fig. 3.** (a) Scatter plot of  $aa_{Lmax}$  against  $aa_{Hmin}$  (triangles) and the linear fit (solid). (b) Scatter plot of  $T_{Hr}$  against  $aa_{Hmax}$  (triangles) and the linear fit (solid).

At the current state, the smoothed monthly mean sunspot number is as low as 3.1 in September 2019. The monthly mean values for the last few months appear in an oscillation around the minimum. So the minimum time of cycle 25 should not be far from September 2019 ( $\pm 8$  months, Du and Wang, 2011). If the minimum time of  $aa$  index is (temporarily) taken as that of sunspot activity with an average delay of 9 months (Legrand and Simon, 1981; Wang and Sheeley, 2009; Du, 2011b), then the peak time of  $aa_{Hmax}(25)$  would be roughly estimated to be around September 2019 +9 (months) +  $T_{Hr}(25) \sim$  August 2025  $\pm 2.0$  (years).

## 4 Discussions and Conclusions

It is well known that the *aa* index is positively correlated to the solar activity ( $R_1$ ). In general, the stronger the solar activity, the higher the (*aa*) geomagnetic activity. But the correlation between *aa* and  $R_1$  is not very strong (Du, 2011b,c). We can only roughly evaluate the strength of geomagnetic activity from the strength of solar (sunspot) activity. In addition, the future solar activity is also unknown at the present time. The relationship between *aa* and  $R_1$  is not a simple linear one (Borello-Filisetti et al., 1992; Mussino et al., 1994; Kishcha et al., 1999; Lockwood et al., 1999; Echer et al., 2004). The *aa* index tends to lag behind  $R_1$  about 2–3 years around a solar cycle maximum (Wang et al., 2000; Echer et al., 2004), and (only) about 1 year around a cycle minimum (Legrand and Simon, 1981; Wang and Sheeley, 2009; Du, 2011b). The complicate relationship between *aa* and  $R_1$  can be better understood by an integral response model (Du, 2011c) that increases the correlation from 0.61 to 0.85.

There are many methods that can be used to predict the maximum amplitude of sunspot cycle ( $R_m$ ), such as 1) statistical methods, employing the relationship between the inter-cycle parameters (Thompson, 1988; Hathaway et al., 1994) or the early rising rate (Thompson, 1988; Cameron and Schüssler, 2008; Du and Wang, 2012); 2) the functional methods, using mathematical functions of a few parameters (Hathaway et al., 1994; Du, 2011d) for extrapolating the following monthly values; 3) the geomagnetic precursor methods (Ohl, 1979; Brown and Williams, 1969; Du et al., 2009), using the geomagnetic activity near the solar minimum; and 4) the solar precursor ones (Schatten et al., 1978; Pesnell and Schatten, 2018), using the previous cycle's polar field.

In contrast, there are less methods found to predict the maximum amplitude of geomagnetic index. Geomagnetic activity forecast has been over the order of hours or days (McPherron, 1999; Abunina et al., 2013). The annual or monthly prediction on the geomagnetic activity is within a limited accuracy (over 20%) due to the irregular variation in the time series (McPherron, 1999; Gordon, 2015). In the earlier years, Kane

(1988) even pointed out that it is impossible to forecast the long-term geomagnetic activity through analyzing the time series of the  $Ap$  and  $aa$  indices (refer also to Gordon, 2015). The geomagnetic activity near the solar minimum or at the decreasing phase of the solar cycle has been widely used to predict the maximum amplitude of sunspot cycle, but were seldom used to predict the maximum amplitude of the geomagnetic activity itself.

In the current work, we analyzed the **highest ( $aa_H$ ) and the lowest ( $aa_L$ ) 3-hourly  $aa$  index in each 3-day-interval, smoothed by 363 days (121 points) to mimic the 13 months smoothing. It is found that the maximum ( $aa_{Hmax}$ ) of  $aa_H$  is well correlated to both the preceding minimum ( $aa_{Hmin}, r = 0.85$ ) of  $aa_H$  and the preceding minimum ( $aa_{Lmin}, r = 0.89$ ) of  $aa_L$  for the 11-year solar cycle. So, these relationships can be used to predict the strength of geomagnetic activity for the ensuing cycle,  $aa_{Hmax}(25) = 80.1 \pm 7.3$  (nT) or  $91.0 \pm 6.5$  (nT), with an average of  $aa_{Hmax}(25) = 85.5 \pm 6.9$  (nT). It implies that the strength of geomagnetic activity for the ensuing cycle 25 would be similar to the average over the past cycles, but higher than that of cycle 24 by about 32%. Certainly, this value is an upper estimate, as cycle 24 has not completely passed and we should check if there is an even smaller value of  $aa_{Hmin}(25)$  or  $aa_{Lmin}(25)$  than that used in the current work (33.24 or 2.94) in the future few months.**

**If using the high correlation between the smoothed monthly mean  $Ap^3$  and  $aa$  indices since 1932,  $r = 0.94$ , and the linear fitting equation of  $Ap$  to  $aa$ ,**

$$Ap = -1.35 \pm 0.17 + (0.694 \pm 0.008)aa, \sigma = 1.2, \quad (6)$$

**the above prediction will be equivalent to  $Ap_{max}(25) = 56.0 \pm 4.8 \pm 1.2$  (nT), here  $\pm 4.8$  is derived from the uncertainty ( $\pm 6.9$ ) of  $aa_{Hmax}(25)$ .**

The well known 'Waldmeier effect' (Waldmeier, 1939) that the rise time of a solar cycle is well anti-correlated to the amplitude has been widely used to estimate the rise and peak times of a solar cycle if the amplitude has been predicted. However, such a correlation is very weak in the  $aa$  geomagnetic index. The rise time of  $aa_{Hmax}(T_{Hr})$

---

<sup>3</sup><http://www.gfz-potsdam.de/en/kp-index>

for the 11-year cycle is found to be weakly anti-correlated to the following maximum ( $aa_{Hmax}$ ),  $r = -0.42$  at the **84% confidence level**. Using this correlation, one could roughly estimate the rise time,  $T_{Hr}(25) = 5.2 \pm 2.0$  (years), and the peak time, August  $2025 \pm 2.0$  (years), of geomagnetic activity for the ensuing cycle 25. Certainly, this estimate is much less reliable than the predictions on the maximum.

According to the analysis above, the following conclusions may be summarized,

1. **The maximum ( $aa_{Hmax}$ ) of the 363-day smoothed highest 3-hourly  $aa$  index in 3-day-interval ( $aa_H$ ) is found to be well correlated to both the preceding minimum ( $aa_{Hmin}$ ,  $r = 0.85$ ) of  $aa_H$  and the preceding minimum ( $aa_{Lmin}$ ,  $r = 0.89$ ) of the 363-day smoothed lowest 3-hourly  $aa$  index in 3-day-interval ( $aa_L$ ) for the 11-year solar cycle.** As a result, the maximum for the ensuing cycle 25 is predicted to be  $aa_{Hmax}(25) = 85.5 \pm 6.9$  (nT), about 32% higher than that of cycle 24. **This value is equivalent to the Ap index of  $Ap_{max}(25) = 56.0 \pm 4.8 \pm 1.2$  (nT).**
2. **The maximum ( $aa_{Lmax}$ ) of  $aa_L$  is also found to be well correlated to the preceding  $aa_{Hmin}$ ,  $r = 0.80$ . Based on this correlation,  $aa_{Lmax}(25)$  is predicted to be  $6.4 \pm 1.2$  (nT).**
3. **The solar activity is much better correlated to the strong geomagnetic activity ( $aa_{Hmax}$ ,  $r = 0.79$ ) than to the weak one ( $aa_{Lmax}$ ,  $r = 0.37$ ).**
4. **The rise time ( $T_{Hr}$ ) is found to be weakly correlated to the following maximum ( $aa_{Hmax}$ ) for the 11-year cycle,  $r = -0.42$  at the **84% confidence level**.** Using this correlation, one could roughly estimate the rise time,  $T_{Hr}(25) = 5.2 \pm 2.0$  (years), and the peak time, August  $2025 \pm 2.0$  (years), of geomagnetic activity for the ensuing cycle 25.

*Acknowledgements.* We are grateful to the anonymous referee for valuable suggestions which improved this manuscript. This work is supported by the National Science Foundation of China (NSFC) through grants 11973058 and 11603040.

## References

- Abunina, M., Papaioannou, A., Gerontidou, M., Paschalis, P., Abunin, A., and more: Forecasting Geomagnetic Conditions in near-Earth space, *Journal of Physics: Conference Series*, 409, id. 012197, 2013.
- 5 Bartels, J.: Discussion of time variations of geomagnetic activity indices Kp and Ap 1932-1961, *Ann. Geophys.*, 19, 1–20, 1963.
- Borello-Filisetti, O., Mussino, V., Parisi, M., and Storini, M.: Long-term variations in the geomagnetic activity level. I - A connection with solar activity, *Ann. Geophys.*, 10, 668–675, 1992.
- 10 Brown, G.M., Williams, W.R.: Some properties of the day-to-day variability of Sq(H), *Planet. Space Sci.*, 17, 455, 1969.
- Cameron, R., Schüssler, M.: A robust correlation between growth rate and amplitude of solar cycles: consequences for prediction methods, *Astrophys. J.*, 685, 1291–1296, 2008.
- Chen, S., Chai, L., Xu, K., Wei, Y., Rong, Z., and Wan, W.: Estimation of the Occurrence Probability of Extreme Geomagnetic Storms by Applying Extreme Value Theory to Aa Index, *J. Geophys. Res.: Space Physics*, 124, 9943–9952, 2019.
- 15 Clette, F., Cliver, E., Lefèvre, L., Svalgaard, L., Vaquero, J., Leibacher, J.: Preface to topical issue: recalibration of the sunspot number, *Solar. Phys.*, 219, 2479–2486, 2016.
- Cliver, E. W., Boriakoff, V., and Feynman, J.: Solar variability and climate change: Geomagnetic aa index and global surface temperature, *Geophys. Res. Lett.*, 25, 1035–1038, 1998.
- 20 Dobrica, V., Demetrescu, C., Boroneant, C., and Maris, G.: Solar and geomagnetic activity effects on climate at regional and global scales: Case study-Romania, *J. Atmos. Sol. Terr. Phys.*, 71, 1727–1735, 2009.
- Du, Z. L.: The correlation between solar and geomagnetic activity – Part 1: Two-term decomposition of geomagnetic activity, *Ann. Geophys.*, 29, 1331–1340, 2011a.
- 25 Du, Z. L.: The correlation between solar and geomagnetic activity – Part 2: Long-term trends, *Ann. Geophys.*, *Ann. Geophys.*, 29, 1341–1348, 2011b.
- Du, Z. L.: The correlation between solar and geomagnetic activity – Part 3: An integral response model, *Ann. Geophys.*, 29, 1005–1018, 2011c.
- 30 Du, Z. L.: The Shape of Solar Cycle Described by a Modified Gaussian Function, *Solar. Phys.*, 273, 231–253, 2011d.
- Du, Z. L., and Wang, H. N.: The prediction method of similar cycles, *Research in Astron.*

- Astrophys., 11, 1482–1492, 2011.
- Du, Z. L., and Wang, H. N.: The relationships of solar flares with both sunspot and geomagnetic activity, *Research in Astron. Astrophys.*, 12, 400–410, 2012.
- Du, Z. L., Li, R., and Wang, H. N.: The Predictive Power of Ohl's Precursor Method, *Astron. J.*, 138, 1998–2001, 2009.
- Du, Z. L., and Wang, H. N.: Predicting the solar maximum with the rising rate, *Science China (Physics, Mechanics & Astronomy)*, 55, 365–370, 2012.
- Echer, E., Gonzalez, W. D., Gonzalez, A.L.C. et al.: Long-term correlation between solar and geomagnetic activity, *J. Atmos. Sol. Terr. Phys.*, 66, 1019–1025, 2004.
- El-Borie, M. A., El-Taher, A. M., Thabet, A. A., and Bishara, A. A.: The impact of asymmetrical distribution of solar activity on geomagnetic indices throughout five solar activity cycles, *Adv. Spa. Res.*, 64, 278–286, 2019.
- Feynman, J.: Geomagnetic and solar wind cycles, 1900-1975, *J. Geophys. Res.*, 87, 6153–6162, 1982.
- Gavrilyeva, G. A., Ammosov, P. P., Ammosova, A. M., Koltovskoi, I. I., and Sivtseva, V. I.: Geomagnetic activity signature in seasonal variations of mesopause temperature over Yakutia, *Proceedings of the SPIE*, 10466, id. 1046670, 2017.
- Gonzalez, W. D., Gonzalez, A. L. C., Tsurutani, B. T. et al.: Solar wind-magnetosphere coupling during intense magnetic storms (1978-1979), *J. Geophys. Res.*, 94, 8835–8851, 1989.
- Gonzalez, W. D., Joselyn, J. A., Kamide, Y. et al: What is a geomagnetic storm?, *J. Geophys. Res.*, 99, 5771–5792, 1994.
- Gordon, R.: Forecasting geomagnetic activity at monthly and annual horizons: Time series models, *J. Atmos. Sol. Terr. Phys.*, 133, 111–120, 2015.
- Hathaway, D. H., Wilson, R. M., and Reichmann, E. J.: The shape of the sunspot cycle, *Solar. Phys.*, 151, 177–190, 1994.
- Hathaway, D. H., and Wilson, R. M.: Geomagnetic activity indicates large amplitude for sunspot cycle 24, *Geophys. Res. Lett.*, 33, L18101, 2006.
- Kane, R. P.: Forecasting geomagnetic activity, *PAGEOPH.*, 126, 85–101, 1988.
- Kishcha, P. V., Dmitrieva, I. V., and Obridko, V. N.: Long-term variations of the solar - geomagnetic correlation, total solar irradiance, and northern hemispheric temperature (1868-1997), *J. Atmos. Sol. Terr. Phys.*, 61, 799–808, 1999.
- Legrand, J. P., and Simon, P. A.: Ten cycles of solar and geomagnetic activity, *Sol. Phys.*, 70, 173–195, 1981.

- Li, Y.: Predictions of the features for sunspot cycle 23, *Sol. Phys.*, 170, 437–445, 1997.
- Lockwood, M., Stamper, R., and Wild, M. N.: A doubling of the Sun's coronal magnetic field during the past 100 years, *Nature*, 399, 437–439, 1999.
- 5 Lukianova, R., Alekseev, G., and Mursula, K.: Effects of station relocation in the aa index, *J. Geophys. Res.*, 114, A02105, 2009.
- Marat, D., Galina, D., Viktor, D., and Anna, D.: Dependence of the F2-layer critical frequency median at midlatitudes on geomagnetic activity, *Sol. Terr. Phys.*, 64, 278–286, 2017.
- Mayaud, P. N.: The aa indices: A 100-year series characterizing the magnetic activity, *J. Geophys. Res.*, 77, 6870–6874, 1972.
- 10 McPherron, R.L.: Predicting the Ap index from past behavior and solar wind velocity, *Physics and Chemistry of the Earth, Part C: Solar, Terrestrial & Planetary Science*, 24, 45–56, 1999.
- Mussino, V., Borello-Filisetti, O., Storini, M., and Nevanlinna, H.: Long-term variations in the geomagnetic activity level Part II: Ascending phases of sunspot cycles, *Ann. Geophys.*, 12, 1065–1070, 1994.
- 15 Ohl, A. I.: A new method of very long-term prediction of solar activity, In NASA. Marshall Space Flight Center Solar-Terrest. Predictions Proc., 2, 258–263, 1979.
- Pesnell, W. D., Schatten, K. H.: An early prediction of the amplitude of Solar Cycle 25, *Solar Phys.*, 293, 112, 1994.
- Prestes, A., Rigozo, N. R., Echer, E., and Vieira, L. E. A.: Spectral analysis of sunspot number and geomagnetic indices (1868-2001), *J. Atmos. Sol. Terr. Phys.*, 68, 182–190, 2006.
- 20 Russell, C. T., and Mulligan, T.: The 22-year variation of geomagnetic activity: Implications for the polar magnetic field of the Sun, *Geophys. Res. Lett.*, 22, 3287–3288, 1995.
- Schatten, K. H., Scherrer, P. H., Svalgaard, L., and Wilcox, J. M.: Using dynamo theory to predict the sunspot number during solar cycle 21, *Geophys. Res. Lett.*, 5, 411–414, 1978.
- 25 Singh, P. R., Tiwari, C. M., Saxena, A. K., Agrawal, S. L.: Quasi-biennial periodicities and heliospheric modulation of geomagnetic activity during solar cycles 22C24, *Phys. Scr.*, 94, 105005, 2019.
- Thompson, R. J.: The rise of solar cycle number 22, *Solar Phys.*, 117, 279–289, 1988.
- Tsurutani, B. T., Gonzalez, W. D., Gonzalez, A. L. C. et al.: Corotating solar wind streams and recurrent geomagnetic activity: A review, *J. Geophys. Res.*, 111, A07S01, 2006.
- 30 Waldmeier, M.: Über die Struktur der Sonnenflecken, *Astron. Mitt. Zrich*, 14, 439–450, 1939.
- Wang, Y. M., Lean, J., and Sheeley, N. R.: The long-term variation of the Sun's open magnetic flux, *Geophys. Res. Lett.*, 27, 505–508, 2000.

Wang, Y. M., and Sheeley, N. R.: Understanding the Geomagnetic Precursor of the Solar Cycle, *Astrophys. J.*, 694, L11–L15, 2009.