

Response to the Comments of Reviewer 1 Regarding the Paper:

"Dependence of the critical Richardson number on the temperature gradient in the mesosphere"  
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*In the paper authors described their simulations of dependence of the critical Richardson number on temperature gradient in the mesosphere. I mainly concerned with the question: What is importance (or application) of this work?*

The critical Richardson number  $Ri_c$  is the criterion for the development of turbulence as a result of the dynamic instability induced by gravity waves. The reviewer emphasizes an uncertainty in the estimate of this parameter (see the reviewer's comment below). We believe that, in this case, the estimate of the  $Ri_c$  value using the new independent approach is very important. Also, for the first time, this new approach makes it is possible to establish and estimate the dependence of the  $Ri_c$  value on the temperature negative gradient. Note that according to our results, the  $Ri_c$  value does not depend on the temperature for  $\partial T/\partial z = 0$ . These new results also broaden our understanding of the role of turbulence in the heat balance in the mesosphere. It is shown by us that the  $Ri_c$  increase with the increasing temperature negative gradient induces a rise in the cooling rate in the mesosphere. Our results can also be used to estimate the diffusion coefficient of the eddy turbulence from the experimental data on the energy dissipation rate of the gravity waves. The total density of the thermosphere depends on this coefficient (see our next comment).

It should be emphasized that the temperature negative gradient is the main feature of the mesosphere. This region is very important because of the transition from the *homosphere* to the *heterosphere*. The former corresponds to the mixing process due to turbulence and the latter corresponds to the diffusive separation of the atmospheric constituents. The maximum turbulence takes place in the mesosphere due to the peak of the gravity wave dissipation.

*From theoretical studies and laboratory experiments, the critical Richardson number ( $Ri_c$ ) has been determined to be approximately, but not precisely equal to 0.25 [e.g., Grachev et al., 2012, and references therein]. For example, Haack et al. [2014] observed large variation of energy dissipation rate for wide range of  $Ri$ -values, i.e. also for  $Ri > 1$ . In regions where  $Ri$  was  $> 1$  and far beyond, turbulent layers have been detected. Thus, the increase of  $Ri_c$  from 0.25 to 0.38 found in this work is by no means crucial, especially if we take into account very limited altitude range (within mesosphere-lower thermosphere region) used by authors for simulations.*

It is well known that there is a problem with determining the  $Ri_c$  value. For example, Weinstock [1978] assumed that the turbulence produced in regions of dynamic instability ( $Ri_c < 0.25$ ) could be transported by turbulent flux into regions of larger  $Ri$  and the  $Ri_c$  mean value might be 0.44.

Galperin et al. [2007] considered a spectral theory of turbulence that accounts for strong anisotropy and waves. They lead to the conclusion that “the effects of the nonstationarity, internal waves, and strong anisotropization preclude the laminarization of turbulence and thus make  $Ri_c$  devoid of its conventional meaning”. However, these authors do not take into account the different types of turbulence and the conditions for their development. For example, they do not distinguish between uniform and localized turbulence. The latter is additionally characterized by the Prandtl number. Also, there are different types of wind shear and their interaction with buoyancy forcing, and the latter depends on the temperature gradient. The observations of unstable layers during the Turbulent Oxygen Mixing Experiment (TOMEX) showed the very large values of the energy dissipation rates (0.9 W/kg) and the eddy diffusion coefficients inferred from these data were found to be 1900 m<sup>2</sup>/s by Bishop et al. [2004]. However, Vlasov and Kelley [2015] showed that these large values cannot be considered as the eddy diffusion coefficient because they do not meet the diffusive criterion, so the eddy scales should be much less than the atmospheric density gradient scale. Turbulence with a large eddy scale sometimes cannot be characterized by  $Ri_c$  because of the complicated transition from turbulent to laminar flow. Our results show that  $Ri_c$  can undoubtedly be applied to uniform turbulence with small eddies corresponding to the eddy diffusion. It should be emphasized that the direct measurements of the  $Ri$  value are impossible and this value is estimated using the experimental data on the temperature and wind velocity. These parameters are very variable during the disturbance and sometimes the observed turbulence does not correspond to the current conditions due to the effect of the previous disturbances, as noted by Bishop et al. [2004].

The vertical size of a few kilometers corresponds to the thickness of the uniform turbulent layers considered by us and corresponds to the experimental data (see, for example, TOMEX). The experimental results presented by Haack et al. [2014] are obtained due to the stratospheric observations of very narrow layers with the typical thickness of 40 m corresponding to the localized turbulence. It is surprising that the reviewer reproached us about the small altitude range but recommends a paper with an altitude range that is smaller by a factor of 50 than the range used by us. In this case, the eddy sizes can be comparable with the thickness of the turbulent layer, and the interaction between the buoyancy force and wind shear can be very complicated. Balsley et al. [2008] showed that  $Ri_c$  can only exist when the scales are small. This is in agreement with our conclusion mentioned above. According to Fig. 9 in Haack et al. [2014], the very large buoyancy frequency and the very small wind shear are observed in these turbulent layers. This can indicate the complicated structure of the wind shear (see, for example, Galperin et al. [2007]). Finally, we consider the uniform turbulence corresponding to the criterion for the eddy diffusion [Vlasov and Kelley, 2015] that is very different from the very thin layers of the localized turbulence where scales can be comparable with the thickness of the layer and  $Ri_c$  may not have physical meaning. Also note that the conditions in the lower stratosphere considered by Haack et al. [2014] are very different from the mesosphere. First of all, the density in this region is larger by three orders of magnitude than the mesospheric density.

The  $Ri_c$  increase from 0.25 to 0.38 increases the turbulent cooling rate by a factor of 1.6 (section 5 in our paper). The coefficient  $b = Ri/(P - Ri)$  ( $P$  is the Prandtl number equal to 1 for the uniform turbulence) with  $Ri = 0.25$  is commonly used in the formula  $K_e = b\varepsilon/\omega_B^2$  to estimate the eddy diffusion coefficient where  $\varepsilon$  is the energy dissipation rate and  $\omega_B$  is the buoyancy frequency. The  $Ri_c$  value increases from 0.25 to 0.38, corresponding to the  $b$  and  $K_e$  value increases by a factor of 2, which means a serious change in the height distributions of the important constituents in the mesosphere and lower thermosphere. The total density in the thermosphere depends on this coefficient because the atomic oxygen height distribution formed by eddy diffusion in the mesosphere becomes the main constituent in the thermosphere.

*List of references contains only 14 items (with 2 from the same authors).*

*That is, a proper overview of previous results regarding  $Ri_c$  is not present. Quick look in 50 years old overview articles [e.g., Reiter and Lester, 1967; Obukhov, 1971] reveals that different values of  $Ri_c$  with different approaches were obtained, which is not mentioned in this manuscript. In the manuscript, authors refer to papers of Miles and Howard from 1961 (i.e., 57 years back). Does it mean, that in the last half century nobody considered this problem?*

This reviewer's statement is incorrect. The references in our paper include, for example:

Abarbanel, H., Holm, D., Marsden, J., and Ratiu, T.: Richardson number criterion for the nonlinear stability of three-dimensional stratified flow, *Phys. Rev. Lett.*, 52, 2352–2355, 1984.

Galperin, B., Sukoriansky, S., and Anderson, P. S.: On the critical Richardson number in stably stratified turbulence, *Atmos. Sci. Lett.*, 8(3), 65-69, 2007.

Hysell, D. F., Nossa, E, Larsen, M, F., Munro, Smith, J S., Sulzer, M. P. and González, S. A.: Dynamic instability in the lower thermosphere inferred from irregular sporadic E layers, *J. Geophys. Res.*, 117, A08305, doi:10.1029/2012JA017910, 2012.

Ligni eres, F., Califano, F., and Mangeney, A.: Shear layer instability in a highly diffusive stably stratified atmosphere, *Astron. and Astrophys.*, 349, 1027-1036, 1999.

As can be seen from this list of references, the papers published in 2007 and 2012 are included. Note that the paper by Hysell et al. [2012] is based on the approach developed by Miles [1961]. It is necessary to emphasize that the goal of our paper is the theoretical estimate of the critical Richardson number for the development of dynamic instability in the mesosphere. The large negative gradient of the temperature is the main feature of this region of the upper atmosphere. However, excluding the paper by Hysell et al. [2012], the other papers mentioned above are not related to the mesospheric condition and this is even more so for the papers mentioned by the reviewer. The latter are dealing with the turbulence in the boundary layer where atmospheric gas interaction with the surface of the earth plays a very important role and the conditions are very different from the conditions in the upper atmosphere. The former is the object of *meteorology*,

which studies the lower atmosphere, and the latter is the object of *aeronomy*, which studies the behavior of the free gas in the upper atmosphere.

It is noted in our paper that “we could not find papers on the theoretical estimate of the critical Richardson number that take the mesospheric conditions (first of all, the large negative gradient of the temperature) into account”. Unfortunately, the reviewer also could not find papers concerning the problem considered in our paper. Three of the four references given by the reviewer are not concerned with the upper atmosphere because these papers are dealing with the boundary layer in the lower atmosphere. The paper [Haack et al., 2014] mentioned by the reviewer in the reviewer’s previous comment was discussed in detail in our response to that reviewer’s comment. Note that Haack et al. [2014] quotes the Galperin et al. [2007] paper mentioned in our paper. Finally, we are very surprised that the reviewer does not make a distinction between the atmosphere in the boundary layer and the upper atmosphere.

*Also some reference to the statement "However, the eddy turbulence peak is observed in the mesosphere or the lower thermosphere where the large negative and positive gradients of the temperature occur." (lines 47-49) is needed.*

*Authors concluded, that  $Ri_c$  value depends on the temperature gradient. The  $Ri_c$  value increases with the negative mesospheric temperature gradient increase. (for example, lines 226-227). These statements were supported by different figures that only show an altitude range above 90 km where temperature gradient is already positive. This is confusing. Is it possible to use altitude range below mesopause for simulations?*

It is well known that the peak of turbulence takes place in the mesosphere and low thermosphere [Brasseur and Solomon, 1986; Fukao, et al., 1994]. First of all, this is a result of the maximum energy dissipation of the gravity waves. The mean energy dissipation rate can be 0.3 – 0.4 W/kg [Bishop et al., 2004] in this region, but this mean rate does not exceed 0.04 W/kg [Haack et al., 2014] in the stratosphere.

The reviewer’s confusion is wrong. At first, this confusion was based on a very old schematic diagram of the temperature height distribution in the upper atmosphere (see, for example, Banks and Kockarts [1973]). However, modern schematic diagrams show the mesopause altitude of 95 km (see, for example, Schunk and Nagy [2009]). According to the global experimental data generalized by the empirical models, the mesopause is located at the altitude significantly above 90 km in winter, spring, summer, and autumn seasons at all latitudes and the maximum of the negative gradient of the temperature takes place at around 90 km, excluding the latitude region of 50°N – 70°N in summer where this altitude can be below 90 km. An example of the latitudinal distributions of the mesopause altitude in the summer and autumn equinoxes is shown in Fig. 1, according to the global empirical model MSISE-90 [Hedin, 1991]. Note that the mesopause altitudes shown in Fig. 1 can be considered as the mean values between the winter (99 km) and summer (97 km) values at the middle and low latitudes. Additionally, this can be seen from the

annual mean temperature height profile shown in Fig. 2. Unfortunately, the reviewer is more familiar with the lower atmosphere than the upper atmosphere.

Second, the negative gradient of the temperature in the turbulent layer can be produced due to gravity waves at any altitude in the mesosphere and lower thermosphere. However, the temperature negative gradient in the undisturbed mesosphere provides a more comfortable condition for the production of this gradient in the dynamic instability.

Third, it is necessary to emphasize that the  $Ri_c$  dependence on the temperature negative gradient is obtained by us without using the density, neutral composition, and other parameters of the mesosphere (formulas (16) and (17) in our paper). This means that this result can be applied at any altitude where the negative gradient of the temperature takes place in the mesosphere and the lower thermosphere with uniform turbulence.

Unfortunately, the reviewer does not discuss the main points of our paper: a new assumption for the  $Ri_c$  estimate, the influence of the temperature negative gradient on the  $Ri_c$  value, and the increase in the cooling rate due to this influence.

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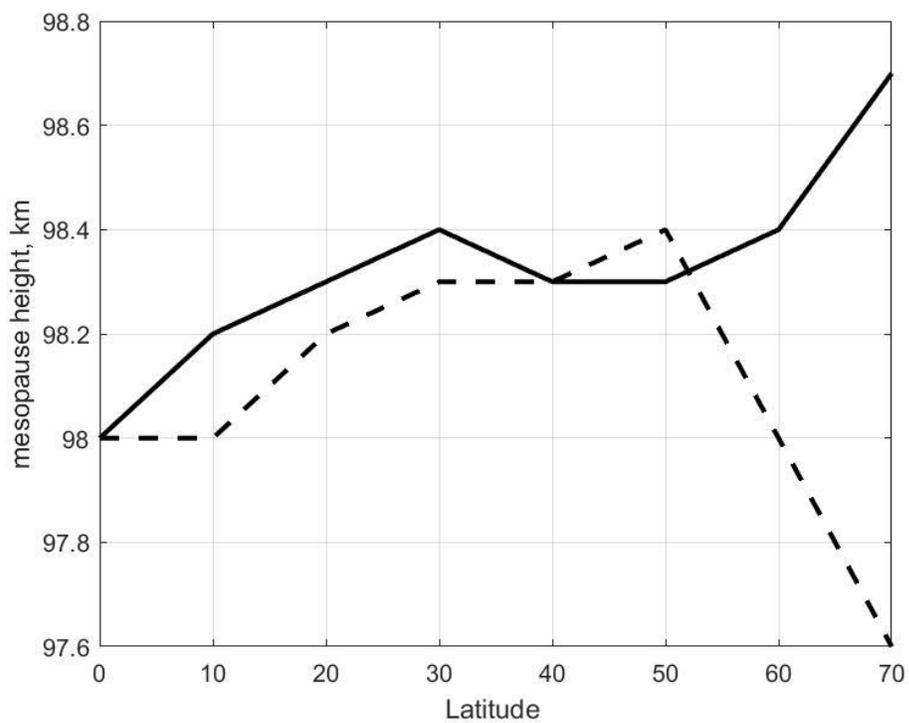


Fig. 1. The latitudinal variations of the mesopause altitude given by the MSISE-90 model [Hedin, 1991] for the autumn and spring equinox (dashed and solid curves, respectively).

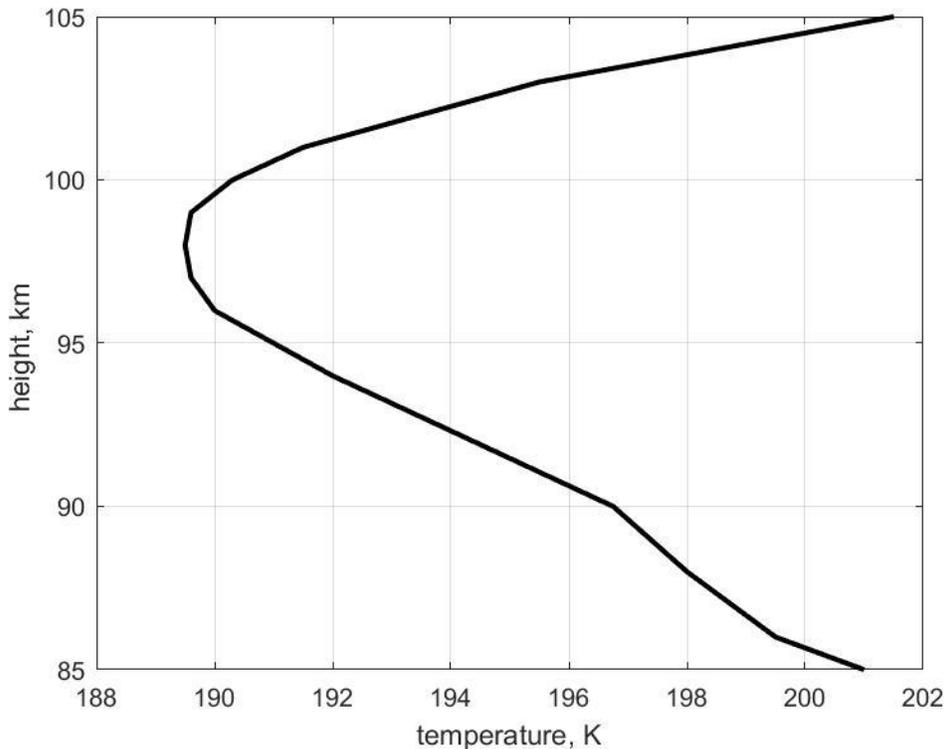


Fig. 2. Annual mean temperature height profile derived from more than 1000 h of Na temperature lidar observations obtained throughout the year and the diurnal cycle at the Urbana Atmospheric Observatory (40°N, 88°W) [Gardner et al., 2002].

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