An objection might be raised that the jump in plasma speed at a boundary between regions of different density, e.g. across either of the two flat boundaries in the slab geometry, is non-physical in that it violate Newton’s laws by not conserving momentum flux across the boundary. We appreciate the opportunity to explain something that may not have been dealt with well in our introduction.

We have taken conservation of particle number into account in Eq. (4), but a careful reader may notice that we have not addressed the conservation of momentum flux across any boundary, and indeed our solutions do not conserve momentum flux within the ionosphere alone.

First, we must orient ourselves and recall that in the ionosphere, plasma drift momentum is fleeting on the timescale of the momentum transfer collision frequencies $\nu_{in}$, and is passed on to the neutral gas. Ion drift, whether across these hypothetical sharp boundaries or in uniformity, is only maintained by the perpendicular electric field (in the frame of the neutrals) as it is in any conducting medium. Rather than being conserved within the plasma there is a continual flow of momentum from the plasma into the neutral gas, and the source of this momentum is the magnetospheric flow which generates the convection electric field in the ionosphere. But the FACs by which this field is sustained can be occurring far outside the area of our study, at much higher and lower electric potentials – there is no requirement for them to be occurring on the boundaries of the very patch we are studying.

We might then further ask, if drift momentum is being drawn from the magnetosphere and deposited in the ionosphere by the FACs, which are carried by electrons of nearly negligible mass, how is this momentum transported perpendicularly to its direction of action, i.e. how is moment conserved? This is explained by the torque acting on a current loop (the ionospheric current closed by the FACs and depolarisation currents in the magnetosphere) within the geomagnetic field.

Second, with this established background field and current in the ionosphere, a conductivity gradient would initially give rise to varied currents. But Ohm’s law per se does not generate electric field structure. It produces charge accumulations and depletions. The electric field structure comes about from Maxwell’s first law, $\epsilon_0 \nabla \cdot \vec{E} = \rho_f - \nabla \cdot \vec{P}$. The RHS is the net charge density.

To see that such net charge densities occur, we can refer e.g. to SuperDARN maps, which show electric potential structure. The conditions being essentially magnetostatic, we have Poisson’s equation, $\epsilon_0 \nabla^2 \phi = -\rho_{net}$. This net charge density might suggest a problem with certain plasma instabilities, however it is orders of magnitude below the charge densities of the ions and electrons themselves.
Once this $\vec{E}$ field structure halts the accumulation, the currents reach steady state with $\nabla \cdot \vec{J} = 0$. In a 2-D plasma, there are no FACs. And we show in App. B that the magnetosphere is sufficiently far that it cannot provide FAC to efface $\vec{E}$ structure on sub-minute scales. Moreover, when an ionospheric structure has had time to begin drawing momentum from the magnetosphere, then it starts to deplete a finite supply, and begins to impress its structure on the magnetospheric flow.

We hope that this explanation serves to answer the concerns about the physicality of our results.